1. Integrate the following

(a) $\displaystyle\int \frac{3x+1}{x^3+2x^2+x}\,dx$

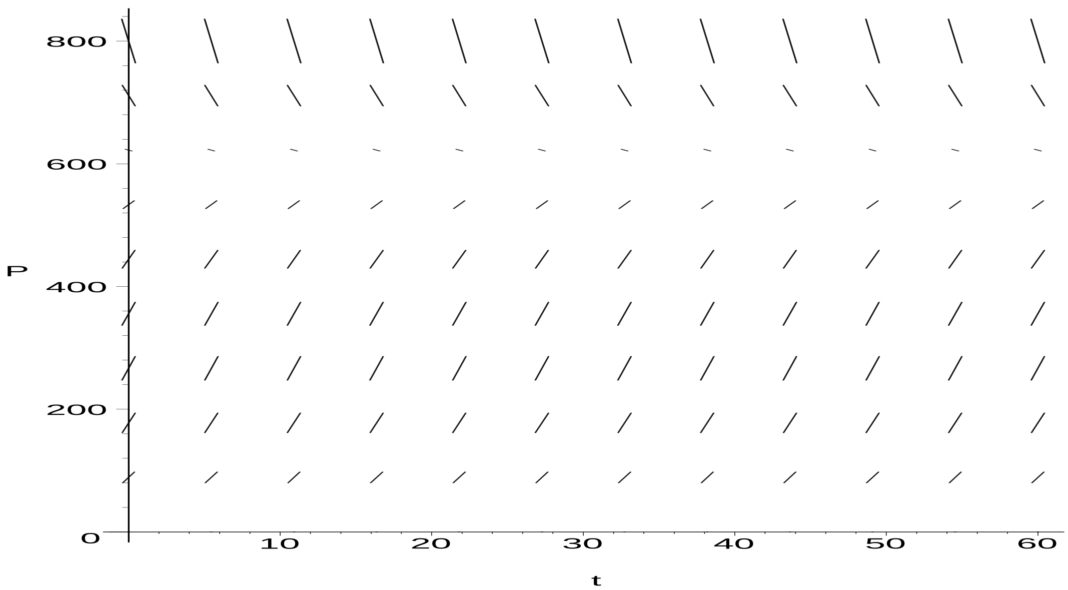
(b) $\displaystyle\int \cos^3 x \sin x\,dx$

(c) $\displaystyle\int \sqrt{x}\ln x\,dx \qquad x > 0$

2. The population of a certain species of fish in a pond can be estimated by the differential equation

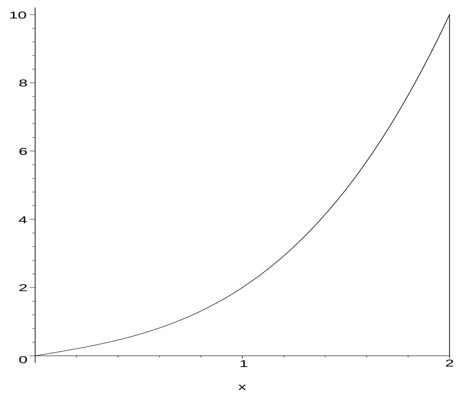
$$\frac{dP}{dt} = .05P(600 - P)$$

and the slope field looks like:




(a) If in the Spring, the pond is initially stocked with 150 fish. What is the maximum capacity of fish in this pond? On the slope field shown, draw the curve of $P(t)$ detailing any changes in concavity and horizontal asymtotes.

(b) Solve this initial value differential equation for $P(t)$.




3. Determine the volume of the solid generated by rotating the region bounded by the $x$-axis, $x = 2$ and $f(x) = x^3 + x$ about the $y$-axis (pictured above).

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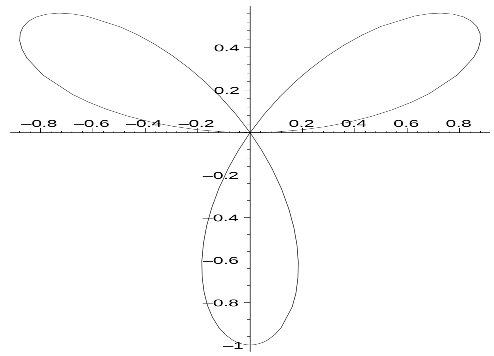
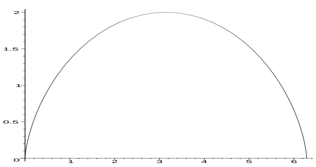



5. Determine the area of one petal of the three-petal rose, $r(\theta) = \sin(3\theta)$ as shown.

6. Determine the tangent vector to this polar curve, $r(\theta) = \sin(3\theta)$ at $\theta = \frac{\pi}{3}$.

7. Determine the arc length of the cycloid on $0 \leq t \leq 2\pi$ described by

$$\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$$



8. Determine the mass and the center of mass of a 2-m rod whose density varies linearly from 3.2 kg/m to 6.8 kg/m.

9. Evaluate the following improper integral if it converges. If it diverges, show reason for divergence.

(a) $\displaystyle\int_0^\infty \frac{4}{x^2+1}\,dx$

(b) $\displaystyle\int_1^4 \frac{4}{(x-1)^2}\,dx$

(c) $\displaystyle\int_0^\infty \frac{4x}{x^2+1}\,dx$

10. Decide whether each of the following infinite series converges or diverges, citing an appropriate test and justifying your conclusion.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{n^2+19n}}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{9^{2n-1}}{4^{4n+2}}$

(d) $\displaystyle\sum_{n=2}^{\infty}\frac{3n-1}{n^3-4}$

(e) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{n^{20}}$

11. Find the interval and the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+3)^n}{(n+1) \cdot 2^n}$.

12. Find the fourth-degree Taylor polynomial for $f(x) = \ln(1+x)$ centered on $a = 3$.

13. Consider the three points $A(3,1,0)$, $B(2,0,-1)$, and $C(0,2,3)$.

(a) Find an equation of the plane determined by $A$, $B$, and $C$.

(b) Find the area of the parallelogram with vertices $A$, $B$, and $C$.

(c) Find the volume of the parallelogram with vertices $A$, $B$, $C$, and $D(-1,0,1)$.

(d) Find the projection of $\vec{AB}$ onto $\vec{AC}$.

(e) Find the angle between $\vec{AB}$ and $\vec{AC}$.

14. The position of a particle at time $t$ is given by $\vec{r}(t)$. Find the velocity, speed and acceleration of the particle at time $t = 1$.

15. Let $L$ be the linear function given by:

$$L\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

and let $S$ be the line described by

$$\vec{r}\,(t) = \langle 5, 0, 2 \rangle + t \cdot \langle 4, 5, 2 \rangle.$$

Find an equation for the line $L(S)$.