1. Integrate the following

   (a) $\int \cos^5 x \sin x \, dx$

   (b) $\int \frac{x-1}{x^2-1} \, dx$

   (c) $\int \frac{x+3}{x^2-1} \, dx$

   (d) $\int \sqrt{x} \ln x \, dx$

   (e) $\int \frac{x}{\sqrt{2+3x^2}} \, dx$

   (f) $\int x \cos x \, dx$

2. Evaluate the definite integrals:

   (a) $\int_3^4 \frac{5x-2}{x^2-4} \, dx$

   (b) $\int_1^{e^2} \frac{\ln^2 x}{x} \, dx$

   (c) $\int_0^{1/2} \frac{1}{1-x^2} \, dx$

   (d) $\int_0^1 x^3 \ln x \, dx$

   (e) $\int_0^1 x\sqrt{1+x^2} \, dx$

   (f) $\int_0^\infty te^{-t} \, dt$

3. Solve the initial value problems:

   (a) $\frac{dy}{dx} = 2x\sqrt{1-y^2}$ $\qquad y(0) = 1/2.$

   (b) $\frac{dy}{dx} = x + xy^2$ $\qquad y(0) = 1$

4. Use the washer method to calculate the volume of the solid of revolution that is generated by revolving the region bounded by the curves $y = x$ and $y = -x^2 + 5x$ about the x-axis.

5. Use the shell method to calculate the volume of the solid of revolution which is generated by the region bounded by the curves $y = e^{-x}$, $x = 0$, and $x = 2$, about the y-axis.

6. Determine the volume V of the solid generated by rotating the region bounded by the curves $y = 0$, $y = x^2 + x$ and $x = 2$ about the x-axis.

7. Find the area A of one petal of the three-petal rose $r(\theta) = \sin 3\theta$.

8. Find the area A of the region R which lies in the first quadrant and is bounded by the polar curves $r(\theta) = \theta$ and $r(\theta) = \sin\theta$.

9. Find the area A and the centroid $(\overline{x}, \overline{y})$ of the infinite lamina that is bounded by the curves $x = 0$, $y = 0$, and $y = e^{-x}$.

10. A spherical tank of radius 10 ft is full of liquid of density ω lb/ft^3. Find the work W done by pumping the liquid to the top of the tank.

11. A right circular tank of radius 4 ft and height 10 ft is half full of liquid whose density is ω lb/ft^3. Find the work W done by pumping the liquid to the top of the tank.

12. Find the arc-length L of the given curve C:

    (a) C is the cycloid $\vec{x}(t) = \langle \cos t + t\sin t, \sin t - t\cos t, 1\rangle$, $0 \le t \le \pi$

    (b) C is the curve given by $\vec{x}(t) = \langle t - \sin t, 1 - \cos t, 2\rangle$, $0 \le t \le \pi$

    (c) C is the planar curve given by $y = 3x^{2/3}$, $0 \le x \le 8$.

    (d) C is the planar curve given by $y = f(x)$, $-2 \le x \le 3$ where

$$f(x) = \begin{cases} x + 2 & x \le 0 \\ x^{3/2} + 2 & x \ge 0 \end{cases}$$

13. Evaluate the improper integral if it converges. If it diverges, show reason for divergence.

    (a) $\displaystyle\int_1^4 \frac{1}{(x-1)^2}\,dx$

    (b) $\displaystyle\int_1^\infty \frac{\ln x}{x}\,dx$

    (c) $\displaystyle\int_e^\infty \frac{1}{x\ln^2 x}\,dx$

(d) $\displaystyle\int_0^{\infty} \frac{x}{x^2+1}\,dx$

(e) $\displaystyle\int_0^{\infty} \frac{x}{(x^2+1)^2}\,dx$

(f) $\displaystyle\int_0^{1} \frac{1}{\sqrt{1-x^2}}\,dx$

14. Determine whether the given infinite series is convergent or divergent. In case of convergence, specify if it is absolute or conditional. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n+2}{n^3+3}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+19n}}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n+1}}{n^2+2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3}}{2n^2+4}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^{24}}$

15. Find the sums of the given convergent series:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(c) $\displaystyle\sum_{n=2}^{\infty} \left(\frac{e}{\pi}\right)$

16. Find the interval of convergence for the following power series:

(a) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{(n+1)3^n}$.

(b) $\sum_{n=1}^{\infty} \frac{n(x+1)^n}{(n+1)!}$.

(c) $\sum_{n=0}^{\infty} \frac{(2x+1)^n}{n+1}$.

17. Find the Maclaurin expansion for $f(x) = xe^x + e^{2x}$.

18. Find the Taylor expansion for $f(x) = e^{3x}$, centered on $a = 2$.

19. Find the Maclaurin expansion, together with its radius of convergence, R, for the given function f:

    (a) $f(x) = \frac{x}{1-x}$

    (b) $f(x) = \frac{1}{(1+x)^2}$

    (c) $f(x) = \ln(1+x)$.

20. Consider the three points $A(3,1,0)$, $B(2,0,-1)$, and $C(0,2,3)$.

    (a) Find an equation of the plane determined by A, B, and C.

    (b) Find the area of the parallelogram with vertices A, B, and C.

    (c) Find the volume of the parallelepiped with vertices A, B, C, and $D(-1,0,1)$.

    (d) Find the projection of $\vec{AB}$ onto $\vec{AC}$.

    (e) Find the angle between $\vec{AB}$ and $\vec{AC}$.

21. Consider the line L which passes through the points $P(-1,4,3)$ and $Q(2,1,3)$. Find:

    (a) An equation for the line L.

    (b) An equation for the line that is perpendicular to L and passes through the point $R(5,-2,2)$.

    (c) An equation for the line that is parallel to L and passes through the point $R(5,-2,2)$.

22. Let T be the linear function given by:

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

and let P be the parallelogram with vertices at $(2,1)$, $(1,4)$, and $(4,8)$. Find the area of the parallelogram $T(P)$.

23. Let T be the linear function given by:

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 2 & 1 & 1 \\ 3 & 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and let L be the line described by

$$\vec{r}\,(t) = \langle 5, 0, -2\rangle + t\langle 4, 5, 2\rangle.$$

Find an equation for the line $T(L)$.

24. Consider the line L_1 described by $\vec{r}(t) = \langle -2t+1, 4t, -3t-8\rangle$, and the line L_2 described by $\vec{s}(t) = \langle t+4, -3t-1, 2t-6\rangle$.

   (a) Show that L_1 and L_2 intersect, and find the point of intersection, $\vec{x_0}$.

   (b) Using part (a), find an equation of the plane containing the lines L_1 and L_2.

25. Consider the curve C, given by $\vec{r}(t) = \langle t\sin t, t\cos t, t\rangle$.

   (a) Find the velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$, for each t.

   (b) Find $v = ||\vec{v}||$ and $a = ||\vec{a}||$, for each t.

   (c) Find the tangential component a_τ and the normal component a_n of $\vec{a}$, for each t.

   (d) Use part (c) to find the curvature $\kappa = \kappa(t)$ for each t.

26. Consider the curve C, given by $\vec{r}(t) = \langle t - \sin t, 1, 1 - \cos t\rangle$, $0 \le t \le 2\pi$. Find:

   (a) $\vec{v}$, $\vec{a}$, $v = ||\vec{v}||$, $a = ||\vec{a}||$, for $0 \le t \le 2\pi$.

   (b) a_τ and a_n, for $0 \le t \le 2\pi$.

   (c) $\kappa = \kappa(t)$, for $0 \le t \le 2\pi$

   (d) $\vec{T}$, $\vec{N}$, and $\vec{B} = \vec{T} \times \vec{N}$