MA1032
Fall 2009 Exam I

Show all work to receive full credit. No calculators are allowed.

1. Write the equation of a line perpendicular to the line $3x + y = -1$ and passing through $(1, 3)$. Put the answer in slope-intercept form.

   \[ y = \frac{1}{3}x + b \]
   \[ 3 = \frac{1}{3}(1) + b \]
   \[ 3 - \frac{1}{3} = b \]
   \[ \frac{8}{3} = b \]

   \[ y = \frac{1}{3}x + \frac{8}{3} \] [5]

2. Find the standard form of the equation, center $(h, k)$, and radius $r$ of the circle whose equation is given by

   \[ x^2 + y^2 - 4x + 12y + 31 = 0. \]

   \[ x^2 - 4x + 4 + y^2 + 12y + 36 = -31 + 4 + 36 \]
   \[ (x-2)^2 + (y+6)^2 = 9 \]

   Standard form: \( (x-2)^2 + (y+6)^2 = 9 \) [4]

   \[ (h, k) = (2, -6) \] [2]

   \[ r = 3 \] [2]
3. Use the graph of \( f(x) \) below to answer the following questions.

![Graph of \( f(x) \)](image)

- **a.** Draw \(-f(x)\) on the set of empty axes above. Make sure to label at least three points.
  \((-2,-3)\), \((-1,2)\), \((1,0)\), \((2,1)\), \((3,1)\) \[6\]

- **b.** List all \( x \)-intercept(s) for the original graph.
  \((1,0)\) \[4\]

- **c.** Is \( f(-1) \) positive or negative?

- **d.** For what value(s) of \( x \) is \( f(x) = -1 \)?

- **e.** Find the interval(s) on which the original graph of \( f \) is increasing.
  \((-\infty, \infty)\) \[4\]

4. A function \( g \) is defined by \( g(x) = \frac{A}{x} + \frac{8}{x^2} \). If \( g(-1) = 0 \), find \( A \).

\[
g(-1) = \frac{A}{-1} + \frac{8}{(-1)^2} = 0
\]

\[
A = -8
\]

\[
\frac{y}{-1} + \frac{8}{1} = 0
\]

\[
A = \frac{8}{1}
\]

\[
A = 8
\]

\[4\]
5. Let \( f(x) = \frac{1}{x^2 - 4} \).

a. Is \( f(x) \) even, odd, or neither? Show proof.

\[
\text{If } f(-x) = f(x), \text{ then } f(x) \text{ is \textbf{even}}.
\]

\[
f(-x) = \frac{1}{(-x)^2 - 4} = \frac{1}{x^2 - 4} = f(x)
\]

\[\text{EVEN} \quad [3]\]

b. State the domain of \( f \).

\[x \neq 2, -2 \quad [3]\]

c. If \( f(x) = 4 \), what is \( x \)?

\[
\frac{1}{x^2 - 4} = 4
\]

\[
x^2 - 4 = \frac{1}{4}
\]

\[
x^2 = \frac{17}{4}
\]

\[
x = \pm \frac{\sqrt{17}}{2}
\]

d. Evaluate \( f(x + 2) \) and simplify.

\[
f(x+2) = \frac{1}{(x+2)^2 - 4}
\]

\[\frac{1}{x^2 + 4x + 4 - 4} = \frac{1}{x^2 + 4x} \quad [4]\]

6. Find the average rate of change of \( f(x) = 2x^2 + 7 \) from 2 to 4.

\[
\text{ARC} = \frac{f(4) - f(2)}{4 - 2}
\]

\[
f(4) = 2(4)^2 + 7 = 39
\]

\[
f(2) = 2(2)^2 + 7 = 15
\]

\[
\text{ARC} = \frac{39 - 15}{4 - 2} = \frac{24}{2} = 12
\]

\[12 \quad [4]\]
7. Let \( f(x) = \begin{cases} x & -4 \leq x < 0 \\ -1 & x \geq 0. \end{cases} \)

a. Graph \( f(x) \) on the provided axes.

![Graph of f(x)](image)

b. State the domain and the range of \( f \) using interval notation.

Domain: \( [-4, \infty) \)  
Range: \( [-4, 0) \)

8. Match each story about a bike ride to one of the graphs (i)–(v), where \( d \) represents \textbf{DISTANCE} (in miles) from home and \( t \) is \textbf{TIME} (in hours) since the start of the ride. (A graph may be used more than once.)

![Graphs (i)–(v)](image)

a. Starts 5 miles from home and rides 5 miles per hour away from home.  

b. Starts 10 miles from home and arrives home one hour later.  

c. Starts 10 miles from home and is halfway home after one hour.  

d. Starts 5 miles from home and is 10 miles from home after one hour.
9. Use the graph of \( f(x) \) below to graph \( g(x) = 2f(x+1) - 3 \) on the provided axes. Label four points on the graph of \( g \).

![Graph of \( f(x) \) and \( g(x) \)]

\[ f(x); (-4,3), (-2,0), (0,6), (2,3) \]
\[ g(x); (-5,2), (-3,0), (1,2), (3,4) \]

10. Dan receives $375 per week for selling new and used cars at a car dealership. In addition, he receives 5% of any sales that he generates.

a. Write a linear equation that relates Dan’s weekly salary \( S \) when he has sales \( x \) dollars.

\[ S = 375 + 0.05x \]

b. What is Dan's salary if his weekly sales was $10,000?

\[ S = 375 + 0.05(10000) \]
\[ S = 375 + 500 \]
\[ S = 875 \]
11. Use the figure below. The value of \(d\) is labeled on the \(x\)-axis. Locate and label the following quantities on the figure below.

- \(g(d)\)
- \(g(-d)\)

12. A page with dimensions 11 inches by 14 inches has a border of uniform width \(x\) surrounding the printed matter of the page (see below).

a. Write a formula for the area, \(A\), of the printed part of the page as a function of the width \(x\), of the border.

\[
A = (11-2x)(14-2x)
\]

b. Find the area of the printed page with a border of 1 inch width.

\[
A(1) = (11-2)(14-2) = 108
\]

\[
108 \text{ in}^2
\]