MA 1032
Exam III
Non-Calculator Section

1. Verify the identity. \( \csc \theta \tan \theta = \sec \theta \). [4]

\[
\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} = \sec \theta \\
\frac{1}{\cos \theta} = \sec \theta
\]

2. Verify the identity. \( \frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \csc \theta \) [4]

\[\begin{align*}
\text{Ex} & \quad \text{p} \quad 449 \\
\sin^2 \theta + (1 + \cos \theta)^2 & \quad \frac{(1 + \cos \theta)(\sin \theta)}{(1 + \cos \theta)(\sin \theta)} \\
\sin^2 \theta + 1 + 2 \cos \theta + \cos^2 \theta & \quad \frac{(1 + \cos \theta) \sin \theta}{(1 + \cos \theta) \sin \theta} \\
2(1 + \cos \theta) & \quad \frac{2(1 + \cos \theta)}{(1 + \cos \theta) \sin \theta} \\
1 & \quad \frac{1}{\sin \theta} \\
2 \csc \theta & = 2 \csc \theta
\end{align*}\]
3. Solve each of the following.

a) \( 4^{2x-1} = 7^x \)  
\[ x = \frac{\ln 4}{\ln 7 - \ln 4} \]  
[4]

b) \( \log_3(x) + \log_3(x-2) = 1 \)  
\[ x = 3 \]  
[4]
4. The point \((-3, 4)\) is on the terminal side of the angle \(\theta\) in standard position. Find the exact values of the following trigonometric functions of \(\theta\).

\[
\begin{align*}
\sin \theta &= \frac{4}{5} \\
\cos \theta &= \frac{-3}{5} \\
\tan \theta &= \frac{-4}{3} \\
\csc \theta &= \frac{5}{4} \\
\sec \theta &= \frac{-5}{3} \\
\cot \theta &= \frac{-3}{4}
\end{align*}
\]
5. Find the exact value of each of the following.

a) \( \sin \left( \frac{3\pi}{4} \right) = \frac{\sqrt{2}}{2} \) \[3\]

b) \( \csc \left( -\frac{5\pi}{2} \right) = -1 \) \[3\]

c) \( \sin^2(12^\circ) + \cos^2(12^\circ) = 1 \) \[3\]

d) \( \tan \frac{\pi}{4} - \sin 30^\circ \)

\( 1 - \frac{1}{2} = \frac{1}{2} \) \[3\]
6. Find the amplitude, period, and phase shift of the following function. Use these values to identify the five key points. Sketch a graph of the function using these five key points. [8]

\[ y = 2 \cos \left( \frac{1}{4}x - \frac{\pi}{2} \right) \]

Amplitude \( \frac{2\pi}{\frac{\pi}{4}} = 8 \) Period \( \frac{a}{\frac{\pi}{4}} = 2\pi \) Phase Shift \( \frac{\pi}{2} \)

1st point \( \left( \frac{\pi}{2}, 2 \right) \)

2nd point \( \left( \frac{3\pi}{2}, 0 \right) \)

3rd point \( \left( \frac{5\pi}{2}, -2 \right) \)

4th point \( \left( 2\pi, 0 \right) \)

5th point \( \left( \frac{7\pi}{2}, -2 \right) \)
7. Use the graph below to find the following:

- The amplitude \[ \frac{3}{\pi/2} \]
- The period \[ \frac{\pi}{2} \]
- An equation for the function \[ y = 3 \sin 4x \]

8. Given that \( \tan \theta = \frac{2}{3} \) and \( \sin \theta < 0 \), find the exact values of the following trigonometric functions of \( \theta \).

\[
\cos \theta = -\frac{3}{\sqrt{13}}, \quad r = -\frac{3\sqrt{13}}{13}, \quad \theta = -\frac{\pi}{4}, \quad r = \sqrt{13}
\]

\[
\cot \theta = \frac{3}{2}
\]
9. Find the exact value of each expression:

(a) $\sin^{-1}(-1) = \frac{-\pi}{2}$

(b) $\cos \left( \cos^{-1} \left( \frac{1}{2} \right) \right) = \frac{1}{2}$

(c) $\tan \left( \sin^{-1} \left( \frac{\sqrt{3}}{2} \right) \right) = \sqrt{3}$

(d) $\sin^{-1} \left( \sin \left( \frac{5\pi}{8} \right) \right) = \frac{3\pi}{8}$
10. Find the inverse function of \( f(x) = 5 \sin x + 2 \)

\[
 f^{-1}(x) = \sin^{-1}\left( \frac{x-2}{5} \right)
\]

11. Find the exact solution for the given equation.

\( 4 \sin^{-1}(x) = \pi \)

\[
 x = \frac{\sqrt{2}}{2}
\]
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1. Logan has a garden in the shape of a sector of a circle; the outer rim of the garden is 25 feet long and central angle if the sector $50^\circ$. She wants to add a 3-foot wide walk to the outer rim; how many square feet of paving blocks will she need to build the walk? [5]
(Hint: Draw a picture)

\[ \#28 \ p425 \]
\[ 78.93 \text{ ft}^2 \]

2. A thermometer reading $24^\circ$ F is brought into a room with a constant temperature of $65^\circ$ F. What will it read after being in the room for 8 minutes?
Assume that the thermometer obeys Newton's law of cooling $U(t) = T + (U_0 - T)e^{kt}$ and that $k = -.0520148$.

\[
U(8) = 65 + (24 - 65)e^{-0.0520148(8)}
\]
\[
= 37.94^\circ
\]
3. The half life of carbon-14 is 5730 years. If a picture supposedly painted by Vermeer (1632-1675) contains 99.5% of its carbon -14, how old is the painting? Is it a fake? [8]

\[ \rho = \rho_0 e^{-kt} \]

\[ \frac{\ln(\frac{1}{2})}{5730} = k \]

\[ 0.995 = 1 e^{\frac{\ln(\frac{1}{2})}{5730} t} \]

\[ 41.44 = t \]

Clearly a fake.
4. At time $t$ hours after taking a cough suppressant, the amount $A$ in mg remaining in the body is given by $A = 10e^{-0.1875t}$

a) What is the initial amount taken? [1]

10 mg

b) How many mg are left after 6 hours? [3]

3.25 mg

c) How long before there is 1 mg left in the body? [3]

12.3 hours