3.1. Slopes and Rates of Change

Consider a graph of a car’s distance from home over a two hours period. (Pay attention that as the car began this trip, it was already 10 miles away from home.)

1. Do you think the car was ever moving backwards during this trip? Do you think the car was ever stopped? If so, when?

2. What was the average velocity for the car for the whole trip? (Incl. Units)

3. What was the average velocity for the car for the first one hour of the trip? (Incl. Units)

4. What was the average velocity for the car for the second hour of the trip? (Incl. Units)

5. Estimate the average velocity for the car during the time interval from \( t = 0.4 \) and \( t = 0.6 \).
6. Explain why this might be a reasonable approximation of the velocity at the time \( t = 0.5 \).

7. When the car’s velocity was the largest? Mark on the graph.

8. At the point you chose in the previous question, use a straight ledge to draw in the line tangent to the graph at that point. Picking two points on the line you have drawn, estimate the slope of the line. What are the units? What you have estimated here?

9. Does it seem like the car even exceeded the speed limit (65 m.p.h.)?

10. During which time intervals the car was speeding?

For the following questions you might find helpful using a calculator.

Consider a car with position given by the formula:

\[
v(t) = 90t^2 - 30t^3 + 5
\]

where \( t \) is time in hours and \( s \) is displacement in miles. We will try to estimate the instantaneous velocity of the car at time \( t = 1 \).

11. Compute the average speed of the car from time \( t = 1 \) to time \( t = 1.5 \) using the formula given (use \( v_{ave} = \frac{f(t_f) - f(t_i)}{t_f - t_i} \)).

12. Compute the average speed of the car from time \( t = 1 \) to time \( t = 1.1 \) using the formula given.

13. Compute the average speed of the car from time \( t = 1 \) to time \( t = 1.01 \) using the formula given.

14. Which of the previous three results should be the best estimate of the velocity at time \( t = 1 \)?