1. (10 points) Determine the inversion sequence of the permutation 8 4 3 7 2 6 1 5 of \{1, 2, \ldots, 8\}.

   Answer: 6 4 2 1 3 2 1 0

2. (10 points) Construct the permutation of \{1, 2, \ldots, 8\} whose inversion sequence is 6, 5, 1, 3, 2, 1, 0, 0.

   Answer: 73654218
3. (10 points) How many permutations of \{1, 2, \ldots, 7\} have exactly 19 inversions?

**Answer:** \( \binom{6}{2} + \binom{3}{1} = 20. \)
4. (10 points) Give an example of a cyclic Gray code of order 4 which is not the reflected Gray code.

Answer: There are many different examples. For example, permute the rows of the cyclic Gray code cyclically, or permute columns in the cyclic Gray code.
5. (5 points) Determine the immediate successor of 1011 in the reflected Gray code of length 4.

Answer: 1001

6. (5 points) Determine the 5-tuple of zeros and ones that is the immediate successor of 00111 in the lexicographical order.

Answer: 01000
7. (5 points) Determine the 6-combination of \(\{1, 2, \ldots, 8\}\) that immediately precedes 1, 2, 3, 4, 7, 8 in the lexicographical order.

Answer: 123468

8. (5 points) Determine the 6-combination of \(\{1, 2, \ldots, 8\}\) that immediately follows 1, 2, 3, 6, 7, 8 in the lexicographical order.

Answer: 124567
9. (10 points) How many reflexive and symmetric relations on a finite set $X$ of 9 elements are there?

**Answer:** $2^{36}$. 
10. (10 points) Evaluate the sum

\[ \sum_{k=1}^{n} k \binom{n}{k}. \]

**Answer:** \( n^{2n-1} \).
11. (10 points) Evaluate the sum

\[ 2 + 2 \binom{n}{1} + \frac{2^3}{3} \binom{n}{2} + \frac{2^4}{4} \binom{n}{3} + \ldots + \frac{2^{n+1}}{n+1} \binom{n}{n}. \]

**Answer:** \[ \frac{3^{n+1} - 1}{n + 1}. \]
12. (10 points) Use the multinomial theorem to expand \((x + y + z)^3\).

Answer:

\[
(x + y + z)^3 = \sum_{a+b+c=3} \frac{3!}{a! b! c!} x^a y^b z^c =
\]

\[
x^3 + y^3 + z^3 + 3xy^2 + 3x^2y + 3xz^2 + 3x^2z + 3yz^2 + 3y^2z + 6xyz.
\]